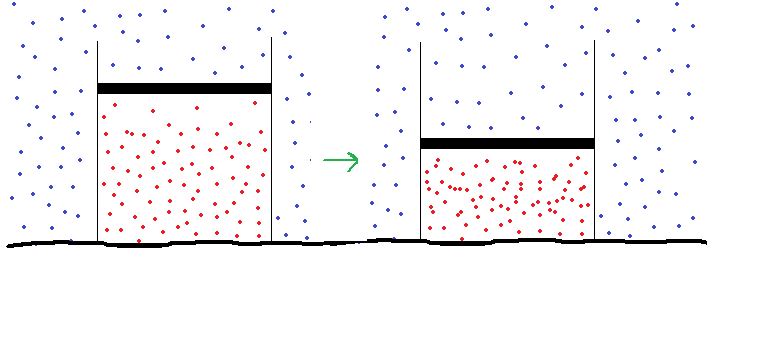
**Various Interacting Mechanical Examples**

Now I’d like to consider several examples of components interacting with each other. The main interest, as before, is to work out the characteristics of the final state. Just to provide an overview of sorts, the basic methodology would be similar to what we did in the previous file. We write down the known initial conditions, as well as known final conditions. And then write down the balance equations of all the components, and solve them as far as possible in terms of left over free state variables **η**. Then we plug them into the expression for the total entropy S(U,V,N;**η**) (where U, V, N are *overall* energy, volume, and particle number, which remains constant presumably) (we must assume that we can write the total entropy as a sum of composite local equilibrium entropies, which seems general enough though, as each component *will* be locally in equilibrium, when the whole system is in equilibrium). Then to determine the equilibrium arrangement, we simply maximize S w/r to **η**. This procedure is quite general in the sense that the **η** can range over quite non-equilibrium values. The **η** do have to be chosen to adequately reflect the real d.o.f. of the system though.

If we want an expression for the time dependence of the system about this equilibrium configuration, we would have to assume that the system components are again in quasi-equilibrium (later called local equilibrium) and that the system’s fluctuations are reversible at this point, which would lead us to the condition. dS/dt = 0 → Σ∂S/∂**η**∙∂**η**/dt = 0. Then we’d plug the (now presumed local equilibrium) balance equations for the **η**’s into this equation. It seems that we also must treat certain **η**’s as correlated now (like assume the volume of the gas has expanded to fill the container, and so that it is correlated with the position/momentum of the lid), since we’re close to equilibrium (but this is no less than what we must do above, to find the equilibrium state). Then dS/dt ought to be zero so we set **F** ≡∂S/∂**η** = 0. This is very similar to the non-equilibrium thermodynamics procedure, except that there we will set **F** proportional to the generalized **J ≡** d**η**/dt. Anyway, evaluating these equations should give us an equation for the reversible fluctuations, such as they are. These **η** can generally just be the individual U, V, N, etc., that come from the component balance equations. And these are the **η** we use when doing the NETD stuff. Note that the totality of conditions on the **η** that will result in d**S**/dt = 0 will also encompass the conditions that will make S maximized (basically setting **F** and **J** both equal to zero will give us the equilibrium conditions). A reduced set of conditions will allow us to describe the approximately reversible fluctuations about equilibrium (basically we *just* set **F** = 0). We need NETD to describe how the system goes from the reduced set of conditions to the entire equilibrium conditions.

**1. Gas and weight, open to environment**

Suppose we have a gas cylinder and we’re supporting a lid on top with mass mℓ. Let’s say it is initially at equilibrium at pgas0, Tgas0, and air also at pair0 , Tair0. And then we let the lid drop on the gas from the initial height, or equivalently, volume Vℓ0 [which is also the initial volume of the gas of course]. If the gas can exchange heat with the outside, what will be the final values of all these things after equilibrium is reestablished? And how would these things change while equilibrium is approached?



To determine where the *interacting* system elements will equilibrate, it is best to perform a balance on all the parts and then fill into the composite entropy balance equation. So we have, for the various components (gas, lid, air):



where Q is the heat exchanged between the gas and the ambient air. And now we fill into the composite entropy balance:



which simply says that the entropy will increase to its maximum value, subject to constraints.

**Equilibrium State**

These are all the equations we have to work out the final state of the system. We can work out most of the unknowns with the entropy equation. We will form an expression for the entropy assuming local compartment equilibrium, and can write the entropy in terms of the independent variables below. We have the following independent variables: Vgas and Vair are independent. Their constraints are Vgas < Vℓ and Vair < V - Vℓ (where V would be, like, the volume of the room basically) as its possible, at this point, that they don’t fill their containers (of course we know they will, but that’s only because we know their entropy equations say they will, technically). Adding up the energy balances we see Ugas + Uair + Elid = U, and that Ugas > 0, Uair > 0, Elid = Pℓ2/2Mlid + mℓgVℓ/A > 0 (‘cause kinetic energy must be `positive, and potential energy minimum would be mg(h = 0)). So any two of the variables Uair, Ugas, Plid may be treated as independent. Well I don’t think so, cause if choose Uair and Ugas, then the energy conservation constraint doesn’t really explicitly come into play. So think going to have to do Uair and Plid or Ugas and Plid. So basically, the maximization of S is taking place on a ‘phase space’ that is complicated by a bunch of physical constraints and balance equation constraints. And I’m effectively choosing independent variables that most simplify extremization over this complicated space. So to start, we have:



Now as mentioned, we maximize w/r to independent variables over the space of local compartment equilibrium functions, meaning we will presume that Sgas and Sair are given by the usual ideal gas entropy equation. We kind of have to do this carefully. For instance, we cannot maximize w/r to Vℓ until we have w/r to Vgas and Vair. I’m guessing, again, that this has to do with the fact that Vgas and Vair cannot vary over *all* values, and in fact, we’re maximizing S over a region of ‘phase space’ which has boundaries that *do* matter. So maximizing w/r to Vgas just says that the gas will fill its volume. And maximizing w/r to Vair says the air will spread to fill *its* volume. So,



where V is total volume. And now we have:



Maximizing w/r to Ugas, Pℓ, and Vℓ, we have:



So for our 6 unknowns: Ugas­, Vgas, Elid, Plid, Uair, Vair, we’ve got, from entropy maximization, the two implicit constraint equations: Vgas = Vℓ, Vair = V - Vℓ, and the three explicit maximization equations above. And we need one more. This can come from overall energy conservation, which follows from the sum of the energy balances. So writing this out, and filling in our entropy maximization results as we go,



Now just have two unknowns. The volume constraint equation is:



Filling this into our energy conservation equation yields:



So now we can solve for pgas, and then get the rest of the guys. This should simplify in the special case where the energy of the air dominates all other energies, so much that it’s properties do not appreciably change so that pair = pair0, Tair = Tair0, etc. Then can say,



which is as we’d expect.

**Fluctuations about Equilibrium**

If we would like to know somewhat the behavior around this equilibrium point, we can do the typical thing which is to fill the balances into dS/dt, set equal to zero to presume reversibility, and then fill back into the balance equations. We will implicitly beforehand presume Vgas and Vair have expanded to fill their max values and so are given by Vℓ and V - Vℓ respectively. And then further, we will therefore be treating Ugas and Uair as independent variables, unlike above, where we treated Ugas and Pℓ as independent.



Might think we should set 1/Tgas – 1/Tair = 0 and (pgas – πgas)/Tgas + (πair – pair)/Tair = 0. But the latter condition violates the 2nd law as applied to the individual compartments. Each compartment separately must statisfy the 2nd law. So we must also write, for the two compartments separately:

So our conditions are:



Note that these conditions encompass our equilibrium state. If we set all of the terms to zero, we’d have: Tgas = Tair, = 0, ℓ = 0 (→ Pℓ = 0) and πgas = pgas, πair = pair. These conditions, in conjunction with the momentum balance on the lid, would imply pgas = pair + mℓg/A, and so we recover all the conditions we derived in the previous section vis a vis the equilibrium state. If we’re close to equilibrium, but not quite there yet, but still presuming reversible fluctuations, then we might just say:



(how we go from this dynamic to ultimately arrive at ℓ = 0 requires NETD) The first row of conditions means isothermal oscillations. The second row means we must be in local equilibrium while oscillating. Filling this into the momentum balance of the lid, whichever its provenance, we have:



Then the energy conservation equation will read:



So we could solve for Tair in terms of Vℓ. Next, pair = NairkTair/(V-Vℓ). And so we can get pair in terms of Vℓ and now we’re all done. Practically speaking, though, Tair and pair would basically be constant, and we can solve the equation treating them so. The equilibrium volume is given by (as this is the volume about which the lid will fluctuate) which implies:



And this is of course our equilibrium condition from before. So looks like treating Ugas, Uair as independent variables, instead of Ugas, Pℓ, works in its own way too. If look for the ODE that governs small oscillations about the equilibrium point, then we’d get:



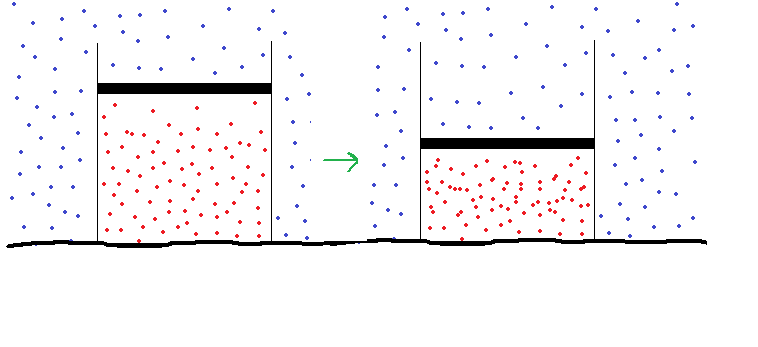
which is simple harmonic motion with period:



but we know that these oscillations would die out necessarily and the volume, pressure would relax to their true equilibrium values. But our equations do not show this because we assumed reversibility.

**2. Gas and weight, thermally isolated from environment**

Suppose we have a gas cylinder with a lid on top with mass mℓ. Let’s say it is initially at equilibrium where p0 = pair, T0 = Tair, and V0. And then we drop the lid on the gas and let it go. If it *can’t* exchange heat with the outside, what will be its new T1, p1, V1, after equilibrium is reestablished? And how would these things change while it approaches equilibrium? Let’s take the gas to be massless for the sake of discussion – doesn’t change anything essential.



We’ll start with balances on the three interacting objects again. These are mostly the same as before.



And now we fill into the composite entropy balance:



which simply says that the entropy will increase to its maximum value, subject to constraints.

**Equilibrium State**

Again we will maximize over the subset of local equilibrium compartments {Vgas, Vair} → Vℓ, Uair, Pℓ, as above. So then we can write the entropy as, noting total conservation of energy, as is apparent from the balance equations themselves.



Maximizing w/r to Vair and Vgas just says that they’ll fill their respective volumes. So,



But, as mentioned before, maximizing over the phase space allowed by physical constraints and our balance equations is somewhat tricky. So in *this* case, on approach to equilibrium, Ugas *won’t* be independent of Vℓ because of the gas energy balance equation (before, the Qga term made these two independent). So maximizing with respect to these variables, using the generic form of S(U,V,N) we would get:



So we now know p1 = pair + mℓg/A, again. But this is all we can say. Unlike before, we cannot ascertain the new temperature. And so we cannot ascertain the new volume per se΄. But, if we make additional assumptions we may. For instance, we can assume that the air constitutes a volume and heat reservoir so that its pressure and temperature don’t really change at all (one implies the other, so we’re really supplying just *one* additional equation relating unknowns). Then, going back to the energy balances, we can fill in the particular energy terms to work out the new volume and temperature.



and referring back to entropy maximization formula we have:



Filling this into the energy balance we have:



and so then the final temperature would be:



**Fluctuations about the Equilibrium State**

If we want to know the approximate time dependence, we will have to make additional assumptions to proceed. So let’s plug the balances into the dS/dt equation and presume reversibility. So we’ll have, implicitly treating {Vgas, Vair} → Vℓ, Ugas, Uair as independent.:



where in the last line we presume the gas has attained the volume of the lid, and the air as well. If we supplement this equation (or really, could replace) with entropy balances on the individual compartments, like we did above, we will see we must set each of the terms separately to zero. So like before we’d get these equations:

So altogether, our conditions would be:



Again, these encompass our equilibrium state, if we set everything to zero: πgas = pgas, πair = pair, and ℓ = 0, ‘cause the latter implies Pℓ = 0, which together with the lid’s momentum balance, implies pgas = pair + mℓg/A. And so we get all of our equilibrium conditions. But we’ll not presume all of these to hold since we don’t want the equilibrium state, but fluctuations about it. So we’ll just say the generalized forces **F** = 0, i.e., πgas = pgas and πair = pair. And then just plug these values into the momentum balance equation:



And then we can say that p = peq­(Veq/V)γ, where Veq is that volume just calculated, and fill this into the equation to get:



This equation is consistent with the former equilibrium calculations since when we set V = Veq., and peq. = pair + mℓg/A, we have , as we would expect. To get the behavior near Veq., we should expand the ODE about this point,



And the solution would be:



So the same as before basically, except for factor of γ. How to see how these fluctuations eventually die down and give V(t) → Veq, we must use NETD.